edexcel
advancing learning, changing lives

Mark Scheme (Final)

Summer 2007

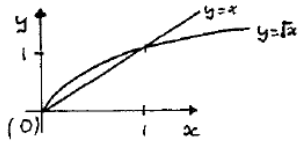
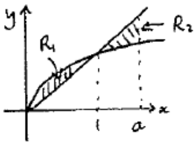
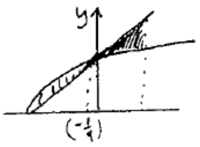
GCE

GCE Mathematics (9801/01)

Edexcel Limited. Registered in England and Wales No. 4496750
Registered Office: One90 High Holborn, London WC1V 7BH

A PEARSON COMPANY

AEA June 2007 – Scheme –

	NOTES	MARKS
1. (a) $(1-y)^{-2} = \underline{1+2y+3y^2+4y^3}$		B1 (1)
(b) Let $y=\cos\theta$, LHS $= \dfrac{1}{(1-\cos\theta)^2}$	Identify $y=\cos\theta$ May be implied	M1
$\cos\theta = 1-2\sin^2\frac{\theta}{2} \Rightarrow 1-\cos\theta = 2\sin^2\frac{\theta}{2}$	$\cos\theta \to \sin\frac{\theta}{2}$	M1
$\therefore \frac{1}{4}\operatorname{cosec}^4\frac{\theta}{2} = 1+2\cos\theta+3\cos^2\theta+\cdots$ (*)	No incorrect work seen	A1 cso.
$\lvert y\rvert<1 \Rightarrow \lvert\cos\theta\rvert<1$ $\therefore$ not valid for $\theta=n\pi$	Accept $0,\pi,2\pi,\ldots$	B1 (4)
(c) $y=\frac{1}{2}$, $\Rightarrow$ Sum $= \dfrac{1}{(\frac{1}{2})^2} = \underline{\underline{4}}$	Attempt $y=\frac{1}{2}$ in LHS o.e. for M1	M1 A1 (2)
(d) $y=-\frac{1}{2}$, $\Rightarrow$ Sum $= \dfrac{1}{(3/2)^2} = \underline{\underline{\frac{4}{9}}}$		M1 A1 (2) (9)
2. (a)  	Relative shapes	B1
	O or (0,0) implied and (1,1) On axes is OK.	B1 (2)
(b) [$R_1=\frac{1}{6}$]  As a increases from 1 R_2 increases. Choose a so that $R_2=R_1$ then areas are the same.	Diagram with regions or mention of areas.	B1g
	Full argument	B1h (2)
(c) $\int_0^a x\,dx = \int_0^a x^{\frac{1}{2}}dx \Rightarrow \left[\frac{x^2}{2}\right]_0^a = \left[\frac{2}{3}x^{3/2}\right]_0^a$	Attempt both integrals - one correct	M1
$\Rightarrow \frac{a^2}{2} = \frac{2}{3}a^{3/2}$	A correct equation in a	A1
$\Rightarrow a^{3/2}(3a^{\frac{1}{2}}-4)=0 \quad \to a^{1/2} = 4/3$ o.e.	Attempt to solve $\Rightarrow a^n = k$	M1
$a = \underline{\underline{\frac{16}{9}}}$		A1 (4)
(d)  Translate $\frac{1}{2}a \leftarrow$.		
$f(x) = \underline{\underline{x+\frac{8}{9}}}$	$x+\frac{a}{2} = f(x)$ (Any suitable $f(x) \pm b$)	B1√
$b = \underline{\underline{\frac{8}{9}}}$	$\frac{a}{2} = b$ √ their a.	B1√ (2) (10)

S.C. if $b=\beta$ and $f(x) = x+\beta$ score B1 only

AEA June 2007 - Scheme | MARKS

(3) (a) $\cos x + \cos 2x = 0 \Rightarrow \cos x + 2\cos^2 x - 1 = 0$ — $\cos 2x \to \cos x \to$ 3TQ — M1

$(2\cos x - 1)(\cos x + 1) = 0 \Rightarrow \cos x = \frac{1}{2}$ or -1 — A1

$\cos x = \frac{1}{2} \Rightarrow x = \underline{\underline{\frac{\pi}{3}}}, \underline{\underline{\frac{5\pi}{3}}}$ — $\alpha,\ 2\pi - \alpha$ — B1, B1√

$\cos x = -1 \Rightarrow x = \underline{\underline{\pi}}$ — B1 — (5)

(Condone degrees but SO)

ALT (a) $\cos x + \cos 2x = 0 \Rightarrow (2)\cos\frac{3x}{2}\cos\frac{x}{2} = 0$ — Use of factor formulae — M1

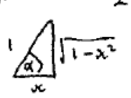
$\Rightarrow \cos\frac{3x}{2} = 0$ or $\cos\frac{x}{2} = 0$ — both — A1

$\cos\frac{3x}{2} = 0 \Rightarrow \frac{3x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots \quad \therefore \quad x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ — B1, B1, B1√

$\left(\cos\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = \frac{\pi}{2}, \ldots \quad \therefore \quad x = \pi\right)$ — (5)

(b) Let: $\arccos x = \alpha$, $\arccos 2x = \beta$

$\therefore \cos(\alpha + \beta) = \cos\frac{\pi}{2} \Rightarrow \cos\alpha\cos\beta - \sin\alpha\sin\beta = 0$ — M1

$\cos\alpha = x$  , $\sin\alpha = \sqrt{1-x^2}$ or $\sin\beta = \sqrt{1-4x^2}$ — M1, A1

or $\cos\beta = 2x$ — A1

$\therefore x \times 2x - \sqrt{1-x^2}\sqrt{1-4x^2} = 0$

$4x^4 = (1-x^2)(1-4x^2)$ — M1

$\Rightarrow 5x^2 = 1$

$x = \underline{\underline{\frac{1}{\sqrt{5}}}}$ — A1 — (6)

ALT 1 (b) $\arccos 2x = \frac{\pi}{2} - \arccos x$

$\therefore (2x =) \cos\left(\frac{\pi}{2} - \arccos x\right) = (\pm)\sin(\arccos x)$ — M1

$\cos\alpha = x$, $\sin(\arccos x) = \sin\alpha = \sqrt{1-x^2}$ — M1, A1

$\therefore 2x = \sqrt{1-x^2}$ or $x = \sqrt{1-4x^2}$ — A1

$\Rightarrow 4x^2 = 1 - x^2 \Rightarrow 5x^2 = 1,\quad x = \underline{\underline{\frac{1}{\sqrt{5}}}}$ — M1, A1 — (6)

ALT 2 (b) First two Ms as before then $2x = \sin\alpha$ (and $x = \cos\alpha$) (M1 M1) A1

$\Rightarrow 2\cos\alpha = \sin\alpha$, $\tan\alpha = 2 \Rightarrow \cos\alpha = ;\ \Rightarrow x = \frac{1}{\sqrt{5}}$ — A1, M1; A1 (6)

Total (11)

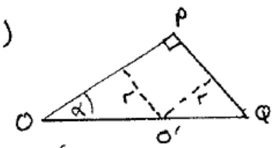
Notes (b)

1st M1	for	use of $\cos(A+B)$ or $\cos\left(\frac{\pi}{2} - \alpha\right) = \pm\sin\alpha$
2nd M1	for	$\alpha = \arccos x \Rightarrow x = \cos\alpha$ (o.e.) and attempt $\sin\alpha$
1st A1	for	$\sin\alpha$ in terms of x ($\sqrt{1-x^2}$ or $2x$)
2nd A1	for	a correct equation in x or α
3rd M1 (dep)	for	reducing equation to $px^2 = q$ or method for value of $\cos\alpha$ (Dependent on both previous Ms)

AEA June 2007 - Scheme

	NOTES	MARKS
4. $\left(\sqrt{\int h(x)dx} =\right) \int \frac{dy}{dx}\, dx = y\ (+c)$	Sub and $\int$ in RHS condone missing $+C$	M1
Square $\quad \int h(x)\, dx = (y+c)^2$	squaring	M1
Differentiate $\quad h(x) = \left(\frac{dy}{dx}\right)^2 = 2(y+c)\cdot\left(\frac{dy}{dx}\right)$	differentiate	M1
$\Rightarrow \quad \frac{dy}{dx} = 2(y+c)$ (*) $\quad \left[\because h>0 \therefore \frac{dy}{dx} \neq 0\right]$	$\div$ by $\frac{dy}{dx}$ [for 5 marks]	M1 A1cso (5)
(b) $\int \frac{1}{y+c}\, dy = 2\int dx$	Separation	M1
$\ln\lvert y+c\rvert = 2x\ (+\alpha)$	correct - condone missing constant	A1
$y+c = Ae^{2x}$	out of logs	M1
$\underline{\underline{y = Ae^{2x} - c}}$ or $Ae^{2x}+k$	A and $\pm c$ needed	A1 (4)
(c) $\frac{dy}{dx} = 2Ae^{2x}$		
$\therefore \quad h(x) = \left(\frac{dy}{dx}\right)^2 = 4A^2e^{4x}$	An expression for h with arbitrary const.	M1
$h(0) = 1 \Rightarrow \quad 4A^2 = 1 \qquad \therefore \underline{\underline{h(x) = e^{4x}}}$	c.s.o.	A1 (2) 11
5. (a) Each $\left(\frac{a}{9}\right)$ square has 3 sides, there are $4\times 3\left(\frac{a}{9}\right)$ squares $\therefore \ 3\times 4\times 3 = 36 \ \left(\frac{a}{27}\right)$ squares.	Convincing argument or calculation. 3×12 or 9×4 OK; 6×6 or 18×2 NOT	B1 (1)
(b) Let P_i = perimeter of S_i $\quad P_1 = 4a, \quad P_2 = 4a + 2\times\frac{a}{3}\times 4 = 4a + \frac{8a}{3} = \frac{20a}{3}$ (*)	Clear counting method	M1
$P_3 = P_2 + 2\times\frac{a}{9}\times 3\times 4 = P_2 + \frac{8a}{3} = \frac{28a}{3}$ (*)	No incorrect work seen	A1 cso (2)
(c) $P_1 = 4a, \quad$ Common difference $= \frac{8a}{3}$	Identify Arithmetic	M1 A1 (both)
$\therefore P_n = 4a + (n-1)\frac{8a}{3}$	Use of n^{th} term formula	M1
$= \underline{\underline{\frac{4a}{3} + \frac{8a}{3}n}}$ or $\underline{\underline{\frac{4a}{3}(2n+1)}}$ or $\underline{\underline{4a + (n-1)\frac{8a}{3}}}$	oe.	A1 (4)
(d) $P_n \to \infty$, as n increases the perimeter $\to \infty$. Accept perimeter increases.		B1 (1)
	(continued over)	

AEA June 2007 – Scheme	NOTES	MARKS
5 (e) Let A_i = area of S_i		
$A_1 = a^2$, $A_2 = \underline{a^2 + 4\left(\frac{a}{3}\right)^2}$ $\left(= a^2 + \frac{4a^2}{9}\right)$	Both A_1, A_2.	B1
$A_3 = A_2 + 4\times3\times\left(\frac{a}{9}\right)^2 = A_2 + \frac{4a^2}{27} = \underline{a^2 + \frac{4a^2}{9} + \frac{4a^2}{27}}$	o.e.	B1 (A_3) (2)
(f) Geometric series : 1st term $\frac{4a^2}{9}$, $r = \frac{1}{3}$	Identify Geometric	M1
	both "a" and r	A1
S_∞ of geometric series is $\dfrac{\frac{4a^2}{9}}{1-\frac{1}{3}}$ $\left(= \frac{2a^2}{3}\right)$	Use of S_∞ formula	M1
$\therefore\ S = a^2 + \text{"}\frac{2a^2}{3}\text{"}$, $= \underline{\underline{\frac{5a^2}{3}}}$	a^2 + their S_∞	M1, A1 (5)
		15
6. (a) $A = \tan\left(\frac{x}{2}\right)\left(\frac{\pi}{2} - x\right)$	o.e.	B1 (1)
(b) $\frac{dA}{dx} = \frac{1}{2}\sec^2\frac{x}{2}\left(\frac{\pi}{2} - x\right) - \tan\frac{x}{2}$	M for use of product rule	M1 A1
N.B. $2\sin x\sec^2\frac{x}{2} = 4\sin\frac{x}{2}\cancel{\cos\frac{x}{2}}\sec^{\cancel{2}}\frac{x}{2} = 4\tan\frac{x}{2}$	Use of $\sin 2A = \ldots$ o.e.	M1 (Must see this)
$\therefore\ \frac{dA}{dx} = \frac{1}{4}\sec^2\frac{x}{2}\left(\pi - 2x - 2\sin x\right)$ (*)	No incorrect working seen	A1 cso (4)
(c) $A'\left(\frac{\pi}{4}\right) = +ve\left(\pi - \frac{\pi}{2} - \frac{2}{\sqrt{2}}\right), = +ve\left(\frac{\pi}{2} - \sqrt{2}\right) > 0$	$A'\left(\frac{\pi}{4}\right)$ attempted > 0	M1 A1 [5 marks for reasons]
$A'\left(\frac{\pi}{3}\right) = +ve\left(\pi - \frac{2\pi}{3} - \sqrt{3}\right), = ve\left(\frac{\pi}{3} - \sqrt{3}\right) < 0$	$A'\left(\frac{\pi}{3}\right)$ attempted < 0	M1 A1
(Change of sign) $\Rightarrow$ stationary point for $\frac{\pi}{4} < x < \frac{\pi}{3}$		M1 A1
$\because$ gradient moves from >0 to <0 $\Rightarrow$ max	convincing argument	A1 (7)
(d) Let $t = \tan\frac{\pi}{8}$, $\left(\tan\frac{\pi}{4} =\right) 1, = \frac{2t}{1-t^2}$	attempt eqn in t	M1
$t^2 + 2t - 1 = 0 \Rightarrow t = \underline{\frac{-2 \pm \sqrt{4+4}}{2}}$	attempt to solve 3TQ	M1
$\left(\because \frac{\pi}{8}\text{ is acute } \therefore t > 0\right) \Rightarrow \underline{t = \sqrt{2} - 1}$ (*)	(for 5 marks)	A1 cso (3)
(e) $A_{max} > A\left(\frac{\pi}{4}\right)$	Attempt @ $A\left(\frac{\pi}{4}\right)$	M1
$\therefore\ A_{max} > \tan\frac{\pi}{8}\left(\frac{\pi}{2} - \frac{\pi}{4}\right)$	No incorrect working seen	A1 cso (2)
i.e. $A_{max} > (\sqrt{2} - 1)\frac{\pi}{4}$		17

AEA June 2007 – Scheme	NOTES	MARKS	
7. (a) $\overrightarrow{OQ}$ is diameter $\therefore\ O\hat{P}Q = 90°$ ($\angle$ in semicircle)	Reason for $O\hat{P}Q = 90°$	B1	
$\therefore\ \underline{p}\cdot(\underline{q}-\underline{p}) = 0$	Use of $\underline{a}\cdot\underline{b}=0$	M1	
$\Rightarrow\ \underline{p}\cdot\underline{q} = \underline{p}\cdot\underline{p} = \lvert\underline{p}\rvert^2$ (*)		A1	(3)
(b) $\overrightarrow{PS} \perp^r \overrightarrow{OQ} \Rightarrow \underline{q}\cdot(\lambda\underline{q}-\underline{p}) = 0$	Full method → equation in λ	M1	
$\therefore\ \lambda \times 9 = \underline{p}\cdot\underline{q} = \lvert\underline{p}\rvert^2 = 6$ $\quad \underline{\underline{\lambda = 2/3}}$		A1	(2)
(c) $\overrightarrow{OR} = \overrightarrow{OP} + 2\overrightarrow{PS}$	A valid vector route for $\overrightarrow{OR}$	M1	
$= \underline{p} + \frac{4}{3}\underline{q} - 2\underline{p} = \frac{4}{3}\underline{q} - \underline{p}$	correct expression	A1	
$\overrightarrow{OR} = \begin{pmatrix} 5/3 \\ -2/3 \\ -5/3 \end{pmatrix}$		A1	(3)
(d) Area of $K = 2\times\Delta OPQ$. $\Delta OPQ = \frac{1}{2}\lvert\underline{q}\rvert\lvert\overrightarrow{PS}\rvert$ or $\frac{1}{2}\lvert\underline{p}\rvert\lvert\overrightarrow{PQ}\rvert$	Formula for area	M1	
$\lvert\underline{p}\rvert = \sqrt{6}$ or $\lvert\underline{q}\rvert = 3$	$\lvert\underline{p}\rvert$ or $\lvert\underline{q}\rvert$	B1	(suitably paired)
and $\lvert\overrightarrow{PQ}\rvert = \left\lvert\begin{matrix}1\\-1\\-1\end{matrix}\right\rvert = \sqrt{3}$ or $\lvert\overrightarrow{PS}\rvert = \left\lvert\begin{matrix}1/3\\-4/3\\-1/3\end{matrix}\right\rvert = \frac{\sqrt{18}}{3}$	$\lvert\overrightarrow{PQ}\rvert$ or $\lvert\overrightarrow{PS}\rvert$	B1	
$\therefore$ Area of $K = 3\times\frac{\sqrt{18}}{3}$ or $\sqrt{6}\times\sqrt{3}$ $= \underline{\sqrt{18} \text{ or } 3\sqrt{2}}$		A1	(4)
(e)  Identify and attempt to use square		M1	
$\tan\alpha = \frac{\lvert\overrightarrow{PQ}\rvert}{\lvert\overrightarrow{OP}\rvert} = \frac{r}{\lvert\overrightarrow{OP}\rvert - r}$			
$(\tan\alpha =)\ \frac{\sqrt{3}}{\sqrt{6}} = \frac{r}{\sqrt{6}-r}$	Forming an equation in r (attempt at)	M1	
	correct	A1	
$\Rightarrow \sqrt{18} - \sqrt{3}r = \sqrt{6}r \quad \therefore\ r = \frac{\sqrt{18}}{\sqrt{6}+\sqrt{3}}$	Obtain expression $r=$	M1	
$= \underline{\sqrt{6}(\sqrt{2}-1)}$	$n = 6$	A1	(5)
(f) Radius of C is $\frac{1}{2}\overrightarrow{OQ} = 3/2$		B1	
Using ratio of area $=$ (ratio of radii)^2.			
Area of $K_1 = \left[\frac{(e)}{3/2}\right]^2 \times (d) = \left(\left[\frac{2\sqrt{6}(\sqrt{2}-1)}{3}\right]^2 \times 3\sqrt{2}\right)$	Full method for area	M1	
$= \underline{\underline{8\sqrt{2}(\sqrt{2}-1)^2 \text{ or } 24\sqrt{2}-32 \text{ or } 8(3\sqrt{2}-4)}}$	or equivalent form with no more surds and simplified fractions	A1	(3)
			(20)

Marks for style clarity and presentation (up to max of 7)

S1 or S2	For a fully correct (or nearly so) and neat or succinct solution to Qu2 – Qu7.	Allow S1 on Qu 1
T1	For a good attempt at whole paper (all questions).	Count best 3

QBA ~~Jun 2007~~
5^{th} July 2007.